$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{encircled}}{\varepsilon_0}$$

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\underbrace{\oint_S \vec{D} \cdot d\vec{s}}_{\text{integral form of gauss' law}} = Q_{encircled} = \int_V \rho \, dV$$

$$\left.\oint_S \vec{D} \cdot d\vec{s} = \int_V (\vec{\nabla} \cdot \vec{D}) \, dV \right\} \text{Vector Calc identity.}$$

$$\int (\vec{\nabla} \cdot \vec{D} - \rho) \, dV = 0$$

$$\left.\therefore \quad \vec{\nabla} \cdot \vec{D} = \rho \right\} \text{local differential form}$$

$$dw = -\vec{F} \cdot d\vec{l} \qquad \vec{F} = q\vec{E}$$

$$w_{AB} = \int_A^B -q\vec{E} \cdot d\vec{l}$$ along the path Γ going from A to B.

$$= -q \int_A^B \vec{E} \cdot d\vec{l}$$

$$V_{AB} = \frac{W_{AB}}{q} = -\int_A^B \vec{E} \cdot d\vec{l}$$

so... to solve a problem we need

* A path $A \to B$
* Need to know $\vec{E}$ everywhere everywhere along the path $A \to B$
* Rule of superposition applies to V b/c it applies to $\vec{E}$ b/c it applies to $\vec{F}$

EX: Place a point charge "q" at the origin.

What is $V_{\infty,x}$ if the path from ∞ to x is along $\hat{x}$

$$\vec{E}(x_p) = \frac{q}{4\pi\varepsilon_0} \frac{\hat{x}}{x^2}$$ for any x_p on the path.

$$d\vec{l} = \hat{x}\, dx_p$$

$$\vec{E} \cdot d\vec{l} = \frac{q}{4\pi\varepsilon_0} \frac{1}{x_p^2} \hat{x} \cdot \hat{x}\, dx_p$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{1}{x_p^2}\, dx_p$$

$$V_{\infty,x} = -\int_\infty^x \vec{E} \cdot d\vec{l} = \frac{-q}{4\pi\varepsilon_0} \int_\infty^x \frac{dx_p}{x_p^2}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{1}{x}$$

In this problem it is convenient to define "the potential at x" as:

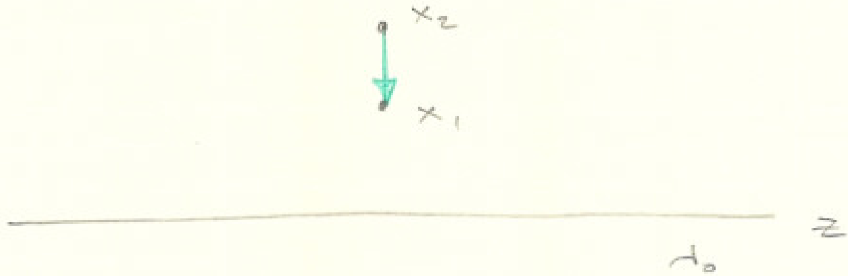
$$V(x) = -\int_{\infty}^{x} \vec{E} \cdot d\vec{l}$$

"b/c" $V(\infty) = 0$; ie- infinity is my refrence and I declare ∞ to be $\emptyset$ potential.

When there are no charges at ∞, declaring $V(\infty)=0$ is very convenient.

EX $\bar{w}$ charges at infinity.

Infinitely long wire $\bar{w}$ charge density λ_0



$$V_{x_2, x_1} = -\int_{x_2}^{x_1} \vec{E} \cdot d\vec{l}$$

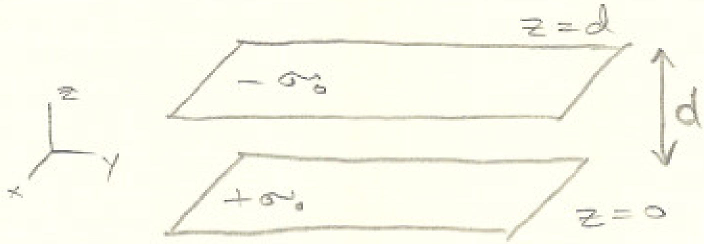
$$= \int_{x_1}^{x_2} \vec{E} \cdot d\vec{l}$$

$$= \int_{x_1}^{x_2} \frac{\lambda_0}{4\pi\varepsilon_0} \frac{\hat{x}}{x_p} \cdot \hat{x} \, dx_p$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0} \ln\left(\frac{x_2}{x_1}\right)$$

note: here if x_2 is ∞, we run into trouble $\ln(\infty)$ is not well defined.

EX:



infinite plates

$$V_{0,d} = -\int_0^d \vec{E} \cdot d\vec{l}$$

$$d\vec{l} = dz_p \, \hat{z}$$

$$\vec{E}_{bottom} = \frac{\sigma_0}{2\varepsilon_0}(+\hat{z})$$

$$\vec{E}_{top} = \frac{(-\sigma_0)}{2\varepsilon_0}(-\hat{z})$$

$$\vec{E} = \frac{\sigma_0}{\varepsilon_0}\hat{z}$$

$$V_{0,d} = -\int_0^d \vec{E} \cdot d\vec{l}$$

$$= -\int_0^d \frac{\sigma_0}{\varepsilon_0}\hat{z} \cdot \hat{z} \, dz_p$$

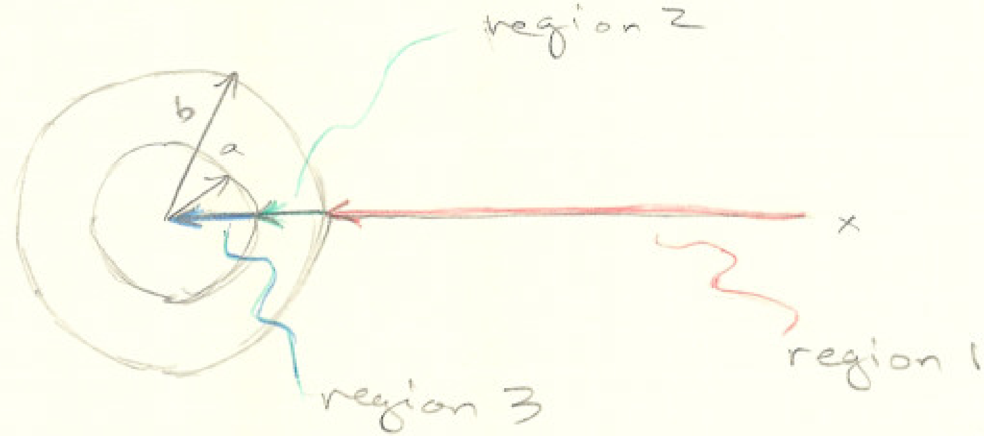
$$= -\frac{\sigma_0}{\varepsilon_0} d$$

note: the feild above and below the plates is zero. (?)

EX: "The Potential" at the center of a hollow sphere of inner radius a and outer radius b carrying constant volume charge ρ_0

Here the path is from ∞ to $x=0$ along $\hat{x}$.

$$d\vec{l} = \hat{x}\, dx_p$$



region 1: use gauss' law to show

$$\vec{E}(x_p) = \frac{Q_{encircled}}{4\pi\varepsilon_0}\frac{\hat{x}}{x_p}$$

$$Q_{encircled} = \int \rho\, dV$$

$$= 4\pi \int_a^b r^2\, dr$$

$$= \frac{4\pi}{3}(b^3 - a^3)$$

$$\vec{E}(x_p) = \frac{4\pi}{3}\rho_0 \frac{(b^3 - a^3)}{4\pi\varepsilon_0}\frac{\hat{x}}{x_p^2}$$

$$= \frac{\rho_0}{3\varepsilon_0}(b^3 - a^3)\frac{\hat{x}}{x_p^2}$$

region 2

$$\vec{E}(x_p) = \frac{1}{4\pi\varepsilon_0}\frac{\hat{x}}{x_p^2}\,\frac{4\pi}{3}\rho_0(x_p^3 - a^3)$$

$$= \frac{\rho_0}{3\varepsilon_0}\left(x_p - \frac{a^3}{x_p^3}\right)\hat{x}$$

region 3:

$$\vec{E}(x_p) = \emptyset$$

Thus $V(0) = -\int_\infty^0 \vec{E}\cdot d\vec{l}$

$$= +\int_0^\infty \vec{E}\cdot d\vec{l}$$

$$= \int_0^a \vec{E}\cdot d\vec{l} + \int_a^b \vec{E}\cdot d\vec{l} + \int_b^\infty \vec{E}\cdot d\vec{l}$$

$$= \frac{\rho_0}{3\varepsilon_0}\left[\int_a^b \left[x_p - \frac{a^3}{x_p^2}\right]dx_p + \int_a^b \left[\frac{b^3 - a^3}{x_p^2}\right]dx_p\right]$$

$$= \frac{\rho_0}{3\varepsilon_0}\left[\left[\frac{x_p^2}{2} + \frac{a^3}{x_p}\right]_a^b + \left[\frac{(b^3 - a^3)}{x_p}\right]_b^\infty\right]$$